A Comparative Study of Optimization Methods for Fuzzy Transportation Problems

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Abstract— Solution of a fuzzy transportation problem determines the transportation schedule that minimizes the total fuzzy transportation cost while satisfying the availability and requirement limits. A number of methods have been devised to solve a fuzzy transportation problem, but the solution obtained by some methods is not optimal. Obviously, the method which gives the optimal solution should be preferred and used in practice.

This work is intended to compare the performance of different methods available for solving a a fuzzy transportation problem and to find out the most appropriate one. For this purpose, a number of fuzzy transportation problems have been solved by different methods and the solution obtained has been tested for optimality. It has been found that the solution obtained by Russell;s Approximation Method is optimal for most of the fuzzy transportation problems.Therefore this method may be regarded as the the most appropriate method for solving a fuzzy transportation problem

Keywords— Fuzzy Transportation Problem, Trapezoidal Fuzzy Number, Ranking Function, Basic Feasible Solution, Optimal Solution

1 INTRODUCTION

THE transportation problem is a special class of linear programming problems in which a commodity is to be transported from various sources of supply to various destinations of demand in such a way that the total transportation cost is a minimum. In general, transportation problem is solved with the assumption that the decision parameters such as availability, requirement and the unit transportation cost are known exactly. But in real life applications, supply, demand and unit transportation cost may be uncertain due to several factors. These imprecise data may be represented by fuzzy numbers.

The transportation problem, in which the transportation costs, supply and demand quantities are represented in terms of fuzzy numbers, is called a fuzzy transportation problem. The objective of the fuzzy transportation problem is to determine the transportation schedule that minimizes the total fuzzy transportation cost while satisfying the availability and requirement limits. Most of the existing techniques provide only crisp solution for fuzzy transportation problem. Chanas etal [1] developed a method for solving fuzzy transportation problems by applying the parametric programming technique using the Bellman–Zadeh criterion [2]. Chanas and Kuchta [3] proposed a method for solving a fuzzy transportation problem by converting the given problem to a bicriterial transportation problem with crisp objective function which provides only crisp solution to the given problem. Liu and Kao [4] proposed a new method for the solution of the fuzzy transportation problem by using the Zadeh's extension principle. Using parametric approach, Nagoorgani and Abdul Razak [5] obtained a fuzzy solution for a two stage Fuzzy Transportation problem with trapezoidal fuzzy numbers. Omar et. alalso proposed a parametric approach for solving transportation problem under fuzziness. Pandian and Natarajan proposed a fuzzy zero point method to find the fuzzy optimal solution of fuzzy transportation problems.

In this paper, the fuzzy transportation problems using trapezoidal fuzzy numbers are discussed. The initial basic feasible solution of the same fuzzy transportation problem is obtained by different methods and then, U-V distribution method is used to find out the optimal solution for the total fuzzy transportation minimum cost.

2 OBJECTIVE

The aim of this paper is to compare the performance of different methods available for obtaining an initial basic feasible solution of the fuzzy transportation problem and to find out the most appropriate one.

3 PRELIMINARIES

3.1 Fuzzy Set

A fuzzy set $\tilde{A}$ on a universal set X is a set of ordered pairs:

$$\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) : x \in X\}$$

3.2 Fuzzy Number

A fuzzy set $\tilde{A}$ defined on the set of real numbers R is said to be a fuzzy number if its membership function $\mu_{\tilde{A}} : R \to [0,1]$ is continuous and such that

$$\mu_{\tilde{A}}(x) = \begin{cases} 0 \quad \forall x \in (-\infty, a] \\ f_A(x) \quad \text{increasing on } [a, b] \\ 1 \quad \forall x \in [b, c] \\ g_A(x) \quad \text{decreasing on } [c, d] \\ 0 \quad \forall x \in [d, \infty) \end{cases}$$

ISSN 2229-5518

where a, b, c ,d are real numbers, and the fuzzy number denoted by $\tilde{A}$ = (a, b, c, d) is called a trapezoidal fuzzy number.

3.3 Trapezoidal Fuzzy Number

A fuzzy number $\tilde{A}$ = (a, b, c, d) is said to be trapezoidal if f_A and g_A are linear functions. The membership function $\mu_{\tilde{A}}$ of a trapezoidal fuzzy number $\tilde{A}$ = (a, b, c, d) is given by

$$\mu_{\tilde{A}}(x)=\begin{cases} 0, & x<a \\ \dfrac{x-a}{b-a}, & a\le x\le b \\ 1, & b\le x\le c \\ \dfrac{d-x}{d-c}, & c\le x\le d \\ 0, & x>d \end{cases}$$

3.4 Ranking Function

Let F(R) denote the set of all trapezoidal fuzzy numbers defined on the set of real numbers R. Then we define a ranking function as a function R : F(R)→R, which maps each fuzzy number into a real number.

If $\tilde{A}$ = (a, b, c, d) , then $R(\tilde{A})=\dfrac{a+b+c+d}{4}$.

For any two trapezoidal fuzzy numbers $\tilde{A}_1=(a_1,b_1,c_1,d_1)$ and $\tilde{A}_2=(a_2,b_2,c_2,d_2)$, we have

(i) $\tilde{A}_1 \le \tilde{A}_2 \Leftrightarrow R(\tilde{A}_1) \le R(\tilde{A}_2)$

(ii) $\tilde{A}_1 \ge \tilde{A}_2 \Leftrightarrow R(\tilde{A}_1) \ge R(\tilde{A}_2)$

(iii) $\tilde{A}_1 \approx \tilde{A}_2 \Leftrightarrow R(\tilde{A}_1) = R(\tilde{A}_2)$

3.5 Arithmetic Operations on Trapezoidal Fuzzy Numbers

Let $\tilde{A}_1=(a_1,b_1,c_1,d_1)$ and $\tilde{A}_2=(a_2,b_2,c_2,d_2)$ be two trapezoidal fuzzy numbers. Then we define

(i) $\tilde{A}_1+\tilde{A}_2=(a_1+a_2, b_1+b_2, c_1+c_2, d_1+d_2)$

(ii) $\tilde{A}_1-\tilde{A}_2=(a_1-d_2, b_1-c_2, c_1-b_2, d_1-a_2)$

(iii) $k\tilde{A}_1 = \begin{cases}(ka_1, kb_1, kc_1, kd_1) & \text{if } k>0 \\ (kd_1, kc_1, kb_1, ka_1) & \text{if } k<0\end{cases}$

(iv) $\tilde{A}_1 . \tilde{A}_2=(a', b', c', d')$

where

$a'=\min(a_1a_2, a_1d_2, a_2d_1, d_1d_2)$, $b'=\min(b_1b_2, b_1c_2, b_2c_1, c_1c_2)$,

$c'=\max(b_1b_2, b_1c_2, b_2c_1, c_1c_2)$, $d'=\max(a_1a_2, a_1d_2, a_2d_1, d_1d_2)$

4 MATHEMATICAL FORMULATION OF A FUZZY TRANSPORTATION PROBLEM

Mathematically, a fuzzy transportation problem can be stated as follows:

$$\text{Minimize } \tilde{Z}=\sum_{i=1}^{m}\sum_{j=1}^{n}\tilde{C}_{ij}\tilde{X}_{ij}$$

subject to the constraints

$$\sum_{j=1}^{n}\tilde{X}_{ij}\approx\tilde{a}_i, \quad i=1,2,\ldots\ldots\ldots\ldots m$$

$$\sum_{i=1}^{m}\tilde{X}_{ij}\approx\tilde{b}_j, \quad j=1,2,\ldots\ldots\ldots\ldots n$$

$$X_{ij}\ge 0$$

where

m = total number of sources

n = total number of destinations

$\tilde{a}_i$ =fuzzy availability of the product at ith source

$\tilde{b}_j$ = fuzzy demand of the product at jth destination

$\tilde{C}_{ij}$ = fuzzy cost of transporting one unit of the product from ith source to jth destination

$\tilde{X}_{ij}$ = fuzzy quantity of the product that should be transported from ith source to jth destination

The above fuzzy transportation problem is said to be balanced if $\sum_{i=1}^{m}\tilde{a}_i=\sum_{j=1}^{n}\tilde{b}_j$, otherwise it is called unbalanced.

5 METHODOLOGY

The solution of a fuzzy transportation problem involves two stages: Finding an initial basic feasible solution, and then, finding the optimal solution from the initial basic feasible solution. In this paper, the same approach has been used to solve a fuzzy transportation problem.

Some fuzzy transportation problems were selected from the literature and their initial basic feasible solution were obtained using different methods. Then, U-V distribution method was used to find out the optimal solution of the test problems, and the results were compared with the true optimal solution.

The brief summaries of three methods, namely North-West Corner Rule, Vogel's Approximation Method[6] and Russell's Approximation Method [7], are given in this paper. The U-V distribution method to find out the optimal solution is also described here.

6 SOLUTION OF A FUZZY TRANSPORTATION PROBLEM

6.1 Finding an Initial Basic Feasible Solution

North-West Corner Rule

The various steps of this method are as follows:

ISSN 2229-5518

Step 1. Select the north-west corner cell of the transportation table and allocate as much as possible so that either the capacity of the first row is exhausted or the destination required of the first column is satisfied, i.e., $\tilde{X}_{11} = \min\left(\tilde{a}_1, \tilde{b}_1\right)$.

Step 2. If $\tilde{b}_1 > \tilde{a}_1$, then move down vertically to the second row and make the second allocation of magnitude $\tilde{X}_{21} = \min\left(\tilde{a}_2, \tilde{b}_1 - \tilde{X}_{11}\right)$ in the cell (2, 1). If $\tilde{b}_1 < \tilde{a}_1$, then move right horizontally to the second column and make the second allocation of magnitude $\tilde{X}_{12} = \min\left(\tilde{a}_1 - \tilde{X}_{11}, \tilde{b}_2\right)$ in the cell (1, 2). If $\tilde{b}_1 = \tilde{a}_1$, then we can make the second allocation of magnitude 0 either in the cell (2, 1) or in the cell (1, 2).

Step 3. Repeat the procedure until all the demands are satisfied.

Vogel's Approximation Method

The various steps of this method are as follows:

Step 1. Find the fuzzy penalties, i.e., the fuzzy difference between the smallest and the next smallest fuzzy costs in each row and column.

Step 2. Select the row or column with the highest fuzzy penalty. If the highest penalties are more than one, choose any one arbitrarily.

Step 3. In the selected row or column found in Step 2, find out the cell having the smallest fuzzy cost. Allocate to this cell as much as possible depending on the fuzzy availability and the fuzzy demands.

Step 4. Delete the row or column which is fully exhausted. For the reduced fuzzy transportation table, again compute the fuzzy penalties, then go to Step 2 and repeat the procedure until all the demands are satisfied.

Russell's Approximation Method

The various steps of this method are as follows:

Step 1. Calculate the quantities $\tilde{U}_i, \tilde{V}_j$ and $\tilde{C}_{ij} - \tilde{U}_i - \tilde{V}_j$ for all i and j , where

$\tilde{U}_i = \max_{1 \le j \le n} \left\{\tilde{C}_{ij}\right\}$ for i = 1, 2,……………….m

$\tilde{V}_j = \max_{1 \le i \le m} \left\{\tilde{C}_{ij}\right\}$ for j = 1, 2,……………….n

Step 2. Select the variables $\tilde{X}_{ij}$ having the most negative value of $\tilde{C}_{ij} - \tilde{U}_i - \tilde{V}_j$. If there are ties in the value of $\tilde{C}_{ij} - \tilde{U}_i - \tilde{V}_j$, select $\tilde{X}_{ij}$ having the smallest unit cost $\tilde{C}_{ij}$. If there are ties again in the value of $\tilde{C}_{ij}$, select $\tilde{X}_{ij}$ having the largest amount of remaining source supply or destination demand.

Set the activity level of $\tilde{X}_{ij}$ equal to the smaller value between the source supply $\tilde{a}_i$ and the destination demand $\tilde{b}_j$.

Step 3. Subtract $\tilde{X}_{ij}$ from $\tilde{a}_i$ or $\tilde{b}_j$ found in Step 3. Eliminate the row or column from the transportation table that results in a zero supply or destination demand after this subtraction. Stop if all $\tilde{a}_i$ and $\tilde{b}_j$ are zero, otherwise go to Step 1.

6.2 Test for Optimality

U-V Distribution Method

The various steps of this method are as follows:

Step 1. Assign a zero trapezoidal fuzzy number to any row or column having maximum number of allocations. If the maximum number of allocations is more than one, choose any one arbitrarily.

Step 2. For each basic cell, find out a set of numbers $\tilde{U}_i$ and $\tilde{V}_j$ satisfying $\tilde{U}_i + \tilde{V}_j = \tilde{C}_{ij}$.

Step 3. For each non basic cell, find out the net evalution $\tilde{U}_i + \tilde{V}_j - \tilde{C}_{ij}$.

Case 1. If $\tilde{U}_i + \tilde{V}_j - \tilde{C}_{ij} < 0$ for all i, j, then the solution is optimal and a unique solution exists.

Case 2. If $\tilde{U}_i + \tilde{V}_j - \tilde{C}_{ij} \le 0$ for all i, j, then the solution is optimal, but an alternate solution exists.

Case 3. If $\tilde{U}_i + \tilde{V}_j - \tilde{C}_{ij} > 0$ for at least one i, j, then the solution is not optimal. In this case, we go to the following step.

Step 4. Select the non basic cell having the largest positive value of $\tilde{U}_i + \tilde{V}_j - \tilde{C}_{ij}$ to enter the basis. Let the cell (r, s) enter the basis. Allocate an unknown quantity, say θ, to the cell (r, s). From this cell (r, s), draw a closed path horizontally and vertically to the nearest basic cell with the restriction that the corner of the closed path must not lie in any non basic cell. Assign signs + and – alternately to the cells of the loop, starting with a + sign for the entering cell.Then θ = minimum of the allocations made in the cells having a negative sign. Add this value of θ to all cells having + sign and subtract the same from the cells having a – sign. Then the allocation of one basic cell reduces to zero.This yields a better solution by making one (or more) basic cell as non basic cell and one non basic cell as basic cell.

Step 5. For the new set of fuzzy basic feasible solution obtained in Step 4, repeat the procedure until a fuzzy optimal solution is obtained.

7 EXAMPLE

For the fuzzy transportation problem given below, find the fuzzy quantity of the product transported from each source to various destinations so that the total fuzzy transportation cost is minimum.

TABLE 1
FUZZY TRANSPORTATION PROBLEM

Dest. (j)→ Source (i)↓	D_1	D_2	D_3	D_4	$\tilde{a}_i$
S_1	(-4,0,4,16)	(-4,0,4,16)	(-4,0,4,16)	(-2,0,2,8)	(0,4,8,12)
S_2	(8,16,24,32)	(8,14,18,24)	(4,8,12,16)	(2,6,10,14)	(4, 8,18,26)
S_3	(4, 8,18,26)	(0,12,16,20)	(0,12,16,20)	(8,14,18,24)	(4,8,12,16)
$\tilde{b}_j$	(2,6,10,14)	(2,2,8,12)	(2,6,10,14)	(2,6,10,14)	(8,20,38,54)

Solution :

Since $\sum_{i=1}^{m} \tilde{a}_i = \sum_{j=1}^{n} \tilde{b}_j$, the problem is a balanced fuzzy transportation problem.

TABLE 2
INITIAL BASIC FEASIBLE SOLUTION BY NORTH-WEST CORNER RULE

Dest. (j)→ Source (i)↓	D_1	D_2	D_3	D_4	$\tilde{a}_i$
S_1	(-4,0,4,16) **(0,4,8,12)**	(-4,0,4,16)	(-4,0,4,16)	(-2,0,2,8)	(0,4,8,12)
S_2	(8,16,24,32) **(-10,-2,6,14)**	(8,14,18,24) **(2,2,8,12)**	(4,8,12,16) **(-22,-6,18,34)**	(2,6,10,14)	(4, 8,18,26)
S_3	(4, 8,18,26)	(0,12,16,20)	(0,12,16,20) **(-32,-12,16,36)**	(8,14,18,24) **(2,6,10,14)**	(4,8,12,16)
$\tilde{b}_j$	(2,6,10,14)	(2,2,8,12)	(2,6,10,14)	(2,6,10,14)	(8,20,38,54)

Since the number of basic cells are m+n-1=6, the solution is non degenerate fuzzy basic feasible solution.

The initial total fuzzy transportation cost is
Minimum Z ($Z^{(1)}$, $Z^{(2)}$, $Z^{(3)}$, $Z^{(4)}$)
= (-4,0,4,16) (0,4,8,12) + (8,16,24,32) (-10,-2,6,14)+ (8,14,18,24)

(2,2,8,12)+(4,8,12,16)(-22,-6,18,34)+(0,12,16,20) (-32,-12,16,36)

+ (8,14,18,24) (2,6,10,14)

≈ (-1328, -200, 972, 2528)

≈ 493

Test for Optimality

Since 2nd row has maximum number of allocations, we take $\tilde{U}_2 = (0,0,0,0)$. Now we compute $\tilde{U}_i$ and $\tilde{V}_j$ for all the basic cells.

We have

$$\tilde{U}_2 + \tilde{V}_1 = \tilde{C}_{21} \Rightarrow (0,0,0,0) + \tilde{V}_1 = (8,16,24,32) \Rightarrow \tilde{V}_1 = (8,16,24,32)$$
$$\tilde{U}_2 + \tilde{V}_2 = \tilde{C}_{22} \Rightarrow (0,0,0,0) + \tilde{V}_2 = (8,14,18,24) \Rightarrow \tilde{V}_2 = (8,14,18,24)$$
$$\tilde{U}_2 + \tilde{V}_3 = \tilde{C}_{23} \Rightarrow (0,0,0,0) + \tilde{V}_3 = (4,8,12,16) \Rightarrow \tilde{V}_3 = (4,8,12,16)$$
$$\tilde{U}_1 + \tilde{V}_1 = \tilde{C}_{11} \Rightarrow \tilde{U}_1 + (8,16,24,32) = (-4,0,4,16) \Rightarrow \tilde{U}_1 = (-36,-24,-12,8)$$
$$\tilde{U}_3 + \tilde{V}_3 = \tilde{C}_{33} \Rightarrow \tilde{U}_3 + (4,8,12,16) = (0,12,16,20) \Rightarrow \tilde{U}_3 = (-16,0,8,16)$$
$$\tilde{U}_3 + \tilde{V}_4 = \tilde{C}_{34} \Rightarrow (-16,0,8,16) + \tilde{V}_4 = (8,14,18,24) \Rightarrow \tilde{V}_4 = (-8,6,18,40)$$

Now we compute the net evaluation $\tilde{U}_i + \tilde{V}_j - \tilde{C}_{ij}$ for all the non basic cells.

We have

$$\tilde{U}_1 + \tilde{V}_2 - \tilde{C}_{12} = (-44,-14,6,36)$$
$$\tilde{U}_1 + \tilde{V}_3 - \tilde{C}_{13} = (-48,-20,0,28)$$
$$\tilde{U}_1 + \tilde{V}_4 - \tilde{C}_{14} = (-52,-20,6,50)$$
$$\tilde{U}_2 + \tilde{V}_4 - \tilde{C}_{24} = (-22,-4,12,38)$$
$$\tilde{U}_3 + \tilde{V}_1 - \tilde{C}_{31} = (-34,-2,24,44)$$
$$\tilde{U}_3 + \tilde{V}_2 - \tilde{C}_{32} = (-28,-2,14,40)$$

Since $\tilde{U}_i + \tilde{V}_j - \tilde{C}_{ij} > 0$ for some i and j , the solution is not optimal

TABLE 3
INITIAL BASIC FEASIBLE SOLUTION BY VOGEL'S APPROXIMATION METHOD

Dest. (j)→ Source (i)↓	D_1	D_2	D_3	D_4	$\tilde{a}_i$
S_1	(-4,0,4,16) **(0,4,8,12)**	(-4,0,4,16)	(-4,0,4,16)	(-2,0,2,8)	(0,4,8,12)
S_2	(8,16,24,32)	(8,14,18,24)	(4,8,12,16) **(-10,-2,12,24)**	(2,6,10,14) **(2,6,10,14)**	(4, 8,18,26)
S_3	(4, 8,18,26) **(-10,-2,6,14)**	(0,12,16,20) **(2, 2, 8, 12)**	(0,12,16,20) **(-22,-6,12,24)**	(8,14,18,24)	(4,8,12,16)
$\tilde{b}_j$	(2,6,10,14)	(2,2,8,12)	(2,6,10,14)	(2,6,10,14)	(8,20,38,54)

Since the number of basic cells are m+n-1=6, the solution is non degenerate fuzzy basic feasible solution.

The initial total fuzzy transportation cost is
Minimum Z ($Z^{(1)}$, $Z^{(2)}$, $Z^{(3)}$, $Z^{(4)}$)
= (-4,0,4,16) (0,4,8,12) + (4,8,12,16) (-10,-2,12,24) + (2, 6,10,14)

(2,6,10,14)+(4,8,18,26)(-10,-2,6,14) +(0,12,16,20) (2,2,8,12)

+(0,12,16,20) (-22,-6,12,24)

≈ (-904, -96, 704, 1856)

≈ 390

Test for Optimality

Since 3rd row has maximum number of allocations, we take $\tilde{U}_3 = (0,0,0,0)$. Now we compute $\tilde{U}_i$ and $\tilde{V}_j$ for all the basic cells.

We have

$$\tilde{U}_3 + \tilde{V}_1 = \tilde{C}_{31} \Rightarrow (0,0,0,0) + \tilde{V}_1 = (4,8,18,26) \Rightarrow \tilde{V}_1 = (4,8,18,26)$$
$$\tilde{U}_3 + \tilde{V}_2 = \tilde{C}_{32} \Rightarrow (0,0,0,0) + \tilde{V}_2 = (0,12,16,20) \Rightarrow \tilde{V}_2 = (0,12,16,20)$$
$$\tilde{U}_3 + \tilde{V}_3 = \tilde{C}_{33} \Rightarrow (0,0,0,0) + \tilde{V}_3 = (0,12,16,20) \Rightarrow \tilde{V}_3 = (0,12,16,20)$$
$$\tilde{U}_2 + \tilde{V}_3 = \tilde{C}_{23} \Rightarrow \tilde{U}_2 + (0,12,16,20) = (4,8,12,16) \Rightarrow \tilde{U}_2 = (-16,-8,0,16)$$
$$\tilde{U}_2 + \tilde{V}_4 = \tilde{C}_{24} \Rightarrow (-16,-8,-0,16) + \tilde{V}_4 = (2,6,10,14) \Rightarrow \tilde{V}_4 = (-14,6,18,30)$$
$$\tilde{U}_1 + \tilde{V}_1 = \tilde{C}_{11} \Rightarrow \tilde{U}_1 + (4,8,18,26) = (-4,0,4,16) \Rightarrow \tilde{U}_1 = (-30,-18,-4,12)$$

Now we compute the net evaluation $\tilde{U}_i + \tilde{V}_j - \tilde{C}_{ij}$ for all the non basic cells.

We have

$$\tilde{U}_1 + \tilde{V}_2 - \tilde{C}_{12} = (-46,-10,12,36)$$
$$\tilde{U}_1 + \tilde{V}_3 - \tilde{C}_{13} = (-46,-10,12,36)$$
$$\tilde{U}_1 + \tilde{V}_4 - \tilde{C}_{14} = (-52,-14,14,44)$$
$$\tilde{U}_2 + \tilde{V}_1 - \tilde{C}_{21} = (-44,-24,2,34)$$
$$\tilde{U}_2 + \tilde{V}_2 - \tilde{C}_{22} = (-40,-14,2,28)$$
$$\tilde{U}_3 + \tilde{V}_4 - \tilde{C}_{34} = (-38,-12,4,22)$$

Since $\tilde{U}_i + \tilde{V}_j - \tilde{C}_{ij} < 0$ for all i and j, the solution is optimal.

For the given fuzzy transportation problem, the initial basic solution obtained by Russell's Approximation Method is exactly the same as that obtained by Vogel's Approximation Method and therefore Russell's Approximation Method also gives a unique optimal solution for the given example.

8 RESULT & DISCUSSION

For the example given here, the initial basic feasible solution obtained by North West Corner Rule is not optimal, whereas the initial basic feasible solution obtained by Vogel's Approximation Method and Russell's Approximation Method is an optimal solution. For some fuzzy transportation problems, it has been found that the initial basic feasible solution obtained by Vogel's Approximation Method is not optimal, but, the initial basic feasible solution obtained by Russell's Approximation Method is optimal. The initial basic feasible solution obtained by North West Corner Rule is far from optimal for most of the fuzzy transportation problems.

The main virtue of the North West Corner Rule is that it is quick and easy. However, because it pays no attention to unit costs (c_{ij}), usually the solution obtained is far from optimal.

Vogel's Approximation Method has been a popular criterion for many years since difference represents the minimum extra unit cost incurred by failing to make an allocation to the cell having the smallest unit cost in that row or column, this criterion does take costs into account in an effective way. Therefore, this method may give an optimal solution in some cases. However, if the solution obtained by this method is not optimal, we have to improve the UV method and continue further till we get an optimal solution.

Russell's Approximation Method is a much more recently proposed criterion that seems very promising. One distinct advantage of Russell's Approximation Method is that it is patterned directly after part-1 of the iterative step for the transportation simplex method which somewhat simplifies the overall computer code.

9 CONCLUSION

For most of the fuzzy transportation problems, the initial basic feasible solution obtained by Russell's Approximation Method is an optimal solution. For some fuzzy transportation problems, the initial basic feasible solution obtained by Vogel's Approximation Method is optimal, but it is not optimal for some other problems. On the other hand, the North-West Corner Rule usually gives the solution far from optimal. Therefore, Russell's Approximation Method is preferred in comparison to other methods for finding the initial basic feasible solution of a Fuzzy Transportation Problem.

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